Influence of Injectant Properties for Fluid-Injection Thrust Vector Control

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A linearized model of fluid-injection thrust vector control is developed. The analysis provides a very simple expression for injectant effective specific impulse and clearly shows effects of injectant and propellant properties on performance. Results compare favorably with a body of gas and liquid injection data available in the open literature. Aerothermochemical aspects are examined by predicting performance of selected injectants in combination with a hypothetical rocket propellant and nozzle. These injectants fall into six classes: inert gases, inert liquids, reactive gases, dissociative liquids, reactive liquids, and liquid bipropellants. Results are discussed in detail.

Nomenclature

A = streamtube cross-sectional area
c_p = constant pressure specific heat of rocket gases (per unit mass)
F = side force
g = gravitational constant
H = energy term defined as $dH = \Delta H_T\,(dw/w) - [(h_{gT} - h_{g0}) + U^2/2](dw_g/w) - [(h_{vT} - h_l) + U^2/2 - V_l^2/2](dw_l/w)$
h = injectant enthalpy (per unit mass)
ΔH = constant-pressure heat of reaction (per unit mass of injectant)
I_{sps} = effective specific impulse of secondary injectant
I_{ds} = density impulse
M = Mach number
$\mathfrak{M}$ = molecular weight
p = static pressure
Q = heat absorbed by injectant (per unit mass) to change from initial temperature to rocket exhaust static temperature
T = temperature
U = velocity of rocket gases, undisturbed
V = velocity of injectant
w = mass flow rate
x, y, z = Cartesian coordinates, x in direction of flow, y normal to wall
α = injection angle; see Fig. 1
β = $1 + (\gamma - 1)M^2$
ν = net moles change of gas in flow system per mole of injectant

Subscripts

g = injectant gas
j = jet
l = injectant liquid
$_0$ = stagnation state
v = vapor
T = at temperature T

Background

Thrust vector control is one of the foremost problems facing the large solid propellant rocket industry. In principle, several methods can be used, but because high-performance propellants produce very high temperatures and usually multiphase flow, thrust vector control methods that minimize contact between moving parts and propellant gases are preferred. At present movable nozzles and fluid injection appear most promising.[1] In this paper we are concerned with aerothermochemical aspects of fluid injection.

Erickson and Bell[2] have summarized many of the previous experimental and theoretical studies of thrust vector control by fluid injection. Experiments are largely uncoordinated, and results have not been correlated with an "all-inclusive" theory—nor are they likely to be, considering the spectrum of experimental variables. Emphasis has been on gas injection, and proposed theoretical models have met with limited success. Liquid injection, complicated by atomization and vaporization, is less amenable to analysis. Since the jet-induced shock wave is strongest in the vicinity of the orifice, theories tend to rely principally on the flow structure near the orifice, with wake and wall effects neglected. Jet expansion, shock-wave boundary-layer interactions, and pressure contours make up the framework of these models.

The present analysis is a simple linearized treatment resulting from a blending of linearized supersonic flow theory with a one-dimensional analysis. The result is a simple expression for the injectant-effective specific impulse which clearly shows aerothermochemical effects.

Analysis

Consider the two-dimensional model shown in Fig. 1. We idealize the problem as constant area mixing between a *trace* of injectant and a portion of the supersonic flow, which is considered an ideal gas. Thermochemical effects (mixing, phase changes, chemical reactions, etc.) are assumed instantaneous and complete. Mixing causes a pressure rise and induces a weak oblique compression wave in the enveloping flow. The transverse component of jet momentum dissi-

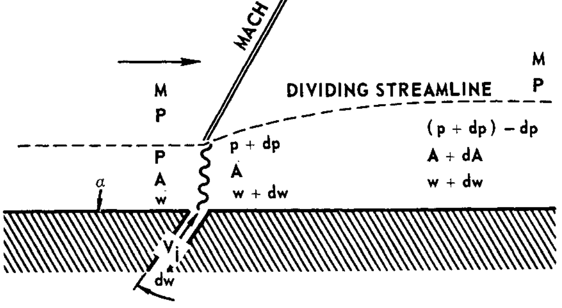

Fig. 1 Linearized model for fluid injection analysis.

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Table 1 Theoretical formulas for induced effective specific impulse for various kinds of injectants

Injectant	$\dfrac{I_{\mathrm{sps},i}}{[pA\gamma M^2/w(M^2-1)^{1/2}]}$
Inert gas	$\dfrac{c_pT_0-(h_{gT}-h_{g0})}{c_pT}+\dfrac{\mathfrak{M}}{\mathfrak{M}_g}-\dfrac{V_g\beta\cos\alpha}{U}$
Inert liquid	$\dfrac{c_pT_0-(h_{vT}-h_l)+v_l^2/2}{c_pT}+\dfrac{\mathfrak{M}}{\mathfrak{M}_l}-\dfrac{V_l\beta\cos\alpha}{U}$
Reactive gas	$\dfrac{c_pT_0-(h_{gT}-h_{g0})+\Delta H_T}{c_pT}+\nu\dfrac{\mathfrak{M}}{\mathfrak{M}_g}-\dfrac{V_g\beta\cos\alpha}{U}$
Reactive liquid	$\dfrac{c_pT_0-(h_{vT}-h_l)+\Delta H_T+V_l^2/2}{c_pT}+\nu\dfrac{\mathfrak{M}}{\mathfrak{M}_l}-\dfrac{V_l\beta\cos\alpha}{U}$

pates in the mixing process. After mixing, the gases expand isentropically until the static pressure equals that of the undisturbed supersonic flow. The ambient flow is displaced with Mach waves maintaining pressure continuity along the dividing streamline.

The side force is composed of two parts: the jet reaction and an induced component, $dF_j + dF_i$.‡

The jet reaction is simply

$$dF_j = (V_j \sin\alpha dw/g) + (p_j - p)dA_j \qquad (1)$$

The induced side force is found by integrating the pressure rise along the dividing streamline. According to two-dimensional linear supersonic flow theory, the pressure coefficient

$$\delta p/(p\gamma M^2/2) = 2(dy/dx)_{\mathrm{str}}/(M^2-1)^{1/2}$$

where $(dy/dx)_{\mathrm{str}}$ is the streamline slope. The induced side force becomes

$$dF_i = \iint (\delta p)dxdz = [\gamma M^2/(M^2-1)^{1/2}]pdA \qquad (2)$$

For isentropic expansion (still linearized)

$$dA/A = [(M^2-1)/\gamma M^2](dp/p) \qquad (3)$$

Pressure rise because of constant area mixing and chemical reaction is easily determined with generalized one-dimensional flow theory as given by Shapiro.[3] From Shapiro's table of influence coefficients, we find§

$$\frac{dp}{p} = \frac{\gamma M^2}{M^2-1}\left[\frac{dH}{c_pT} + 2\left(1+\frac{\gamma-1}{2}M^2\right)\frac{dw}{w} - \frac{V_j\beta\cos\alpha}{U}\frac{dw}{w} - \frac{d\mathfrak{M}}{\mathfrak{M}}\right] \qquad (4)$$

Combining Eqs. (2), (3), and (4), we get

$$I_{\mathrm{sps},i} = \frac{dF_i}{dw} = \frac{pA\gamma M^2}{w(M^2-1)^{1/2}}\left[\frac{w}{c_pT}\frac{dH}{dw} + 2\left(1+\frac{\gamma-1}{2}M^2\right) - \frac{V_j\beta\cos\alpha}{U} - \frac{w}{\mathfrak{M}}\frac{d\mathfrak{M}}{dw}\right] \qquad (5)$$

where $I_{\mathrm{sps},i}$ is the effective specific impulse (induced) of the secondary injectant. Equation (5) is general and is easily reduced for special cases of interest; formulas for gases and liquids, inert or reactive, are given in Table 1. A similar formula for liquid bipropellants is easily established.[4] (Reactive injectants are species that chemically react with mainstream components.) In the Table 1 formulas, ν is the net moles change of gaseous species added to the system per mole of injectant; ν equals 1 for inert gases and vaporized inert liquids but is usually different from unity with chemical change. The terms $(h_{gT} - h_{g0})$, $(h_{vT} - h_l)$, $(h_{gT} - h_{g0}) - \Delta H_T$, and $(h_{vT} - h_l) - \Delta H_T$ are enthalpy changes per unit mass of injectant for the various mechanisms. We call these the Q values for the reaction (positive when injectant absorbs heat and negative when heat is released).

Equation (5) can be developed in closed form only for two-dimensional flow. To obtain a similar solution for three-dimensional flows, approximations are required.[4] The two- and three-dimensional solutions differ by a proportionality constant, which we surmise to be a consequence of the approximations. We will use Eq. (5) to analyze data and make predictions, regardless of injection configuration.

Real gas effects can be included in Eqs. (3) and (4) by using a modified table of influence coefficients given by Cordullo.[5]

The injectant specific impulse can be written

$$I_{\mathrm{sps}} = I_{\mathrm{sps},j} + I_{\mathrm{sps},i} \qquad (6)$$

where

$$I_{\mathrm{sps},j} = (V_j \sin\alpha/g) + (p_j - p)(dA_j/dw)$$

Comparison with Experiment

A proper comparison between linear theory and experiment is provided by extrapolating experimentally measured I_{sps} values to zero injection rate. This way, nonlinear effects are reduced. In addition, the injection port location should preclude (or at least minimize) losses accruing from reflections of the induced shock wave.

We have reported a variety of gas injection TVC data[6, 7] with which linear theory can be checked. In these experiments, two propellants were used (decomposed 90% H_2O_2

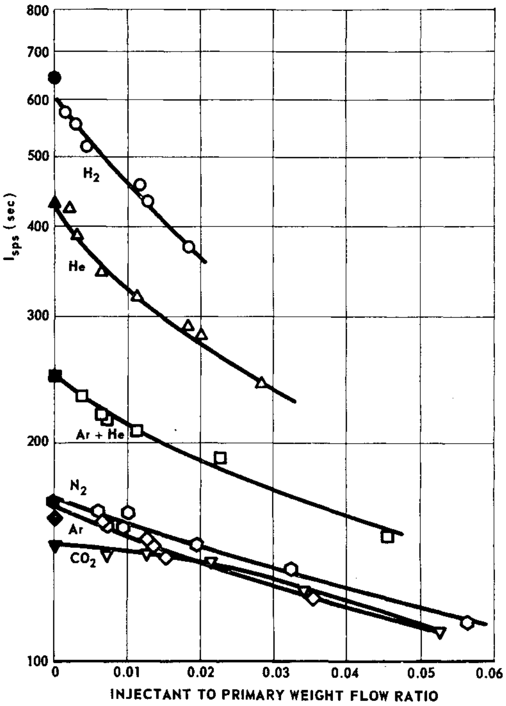

Fig. 2 APL cold-gas injection data—H_2O_2 propellant, M = 1.86 duct flow, α = 90° (solid points are predicted values).

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‡ This reasoning differs from that in an earlier reference by the present authors where the jet reaction was excluded.[4] Certain experimental evidence supports the current reasoning and is discussed later in this paper.

§ Equation (4) is not so general as that given by Shapiro. We exclude heat added from external sources, wall shearing stress, external work, and drag from stationary immersed bodies—that is, $dQ = dW_x = 4f(dx/D) = dX = 0$ in Shapiro's nomenclature.

Table 2 Comparison of Applied Physics Lab. gas injection thrust vector control measurements with linear theory (for H_2O_2 propellant, T_0 = 1850°R; for air propellant, T_0 = 530°R)

A. Conical rocket nozzle data; $M = 2.4$, $T_{0g} = 530°R$

Propellant-Injectant	I_{sps}, sec Expt.[a]	I_{sps}, sec Theo.[b]	Ratio
H_2O_2			
—H_2	515	636	0.81
—He	320	433	0.74
—0.8 He + 0.2 Ar	208	244	0.85
—N_2	140	167	0.84
—Ar	138	158	0.87
—CO_2	130	147	0.88
Air			
—N_2	121	147	0.82
—CO_2	114	128	0.89

B. Supersonic duct flow, $M = 1.9$[c]

Propellant-injectant	T_{0g}	I_{sps}, sec Expt.[a]	I_{sps}, sec Theo.[b]	Ratio
H_2O_2				
—H_2	530	598	645	0.93
—He	530	416	450	0.92
−0.8 + 0.2Ar	530	232	249	0.93
—N_2	530	161	164	0.98
—Ar	530	155	157	0.99
—CO_2	530	145	145	1.00
$-0.78N_2 + 0.19O_2 + 0.03H_2O$[d]	990	175	185	0.95
$-0.77N_2 + 0.18O_2 + 0.05H_2O$[d]	1360	201	219	0.92
$-0.76N_2 + 0.17O_2 + 0.07H_2O$[d]	1660	220	238	0.92
$-0.90H_2 + 0.10H_2O$[e]	1700	890	822	1.08
$-0.81H_2 + 0.19H_2O$[e]	1940	830	697	1.19
$-0.73H_2 + 0.27H_2O$[e]	2200	776	653	1.19
Air				
—N_2	530	121	142	0.85
H_2O_2				
—N_2[c]	530	170	173	0.98

[a] Sonic I_{sps} data extrapolated to zero injection rate.
[b] Theory assumes $p = p_j$.
[c] $M = 2.9$ for last entry (H_2O_2-N_2).
[d] Preburned air with H_2 trace (vitiated air).
[e] Preburned H_2 with O_2 trace (vitiated H_2).

at T_0 = 1850°R or air at T_0 = 530°R). Nozzles were either 15° conical or Clippinger-contoured; the latter provided uniform supersonic duct flow with M = 1.9 or 2.9. Several ambient-temperature injectants and hot reactive and inert injectants were studied. Generally, injection was sonic. Theory and experiment are compared with all of the data in Table 2 and part of the data in Figs. 2, 3, and 4. The supersonic duct flow data are in very good agreement with predictions; these data were the most accurate and the most thoroughly studied. Predictions for the conical nozzle data are high—10–20%—and no explanation can be offered. Injectant properties seem rather well accounted for. One even concludes that H_2-containing injectants, which have the potential of chemically reacting with O_2-rich rocket exhaust, behave almost as inert injectants for these experimental conditions.[¶]

Figure 4 accents injectant velocity effects. All sonic injection data correlate and extrapolate smoothly to zero injection rate (regardless of orifice size represented by different symbols). All subsonic data project to a common but different interception point at zero injection rate. The difference (ΔI_{sps}) is about the same as one might predict. These data support the assumptions for Eq. (1).

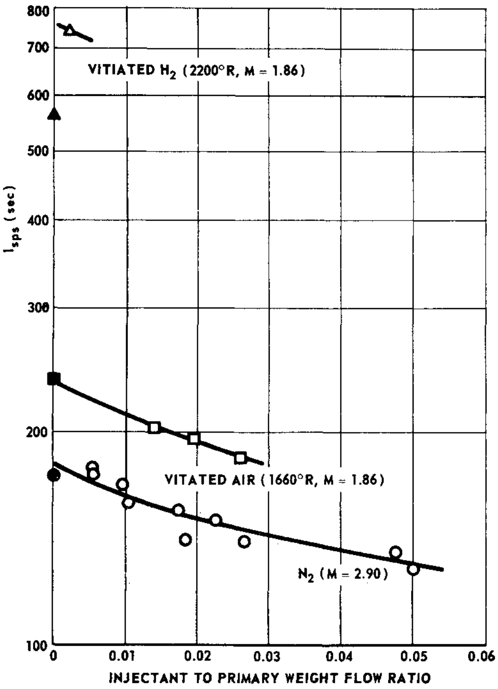

Fig. 3 APL hot-gas injection data—H_2O_2 propellant, M = 1.86 duct flow, α = 90°; cold-gas injection data—H_2O_2 propellant, M = 2.90 duct flow (solid points are predicted values).

Liquid injection data are not so generally available as are gas injection data. Recently, Newton and Spaid[8] reported measurements for Freon 12 injection into a solid propellant rocket exhaust. Figure 5 shows some of these data, which extrapolate fairly well to our predicted value of I_{sps} = 156 sec. Additional data at very low injection rates would have been helpful. Liquid injection data of Green and McCullough[9] also are at high injection rates and make extrapolation difficult; their data for H_2O, Br_2, and Freon 12 are shown in Fig. 6. Predicted values are reasonably consistent with the measurements.

Thermochemical Effects

Many scientists think that better fluid-injection systems can be achieved by exploiting the chemical aspects of the problem. Since linear theory provides a tool with which to judge the potential of an injectant, it seems worth while to

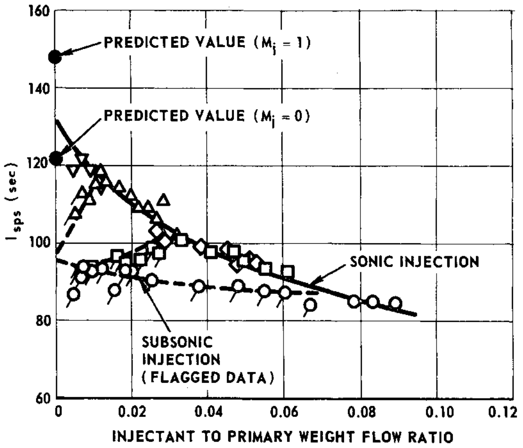

Fig. 4 APL data for sonic-subsonic CO_2 injection in a conical nozzle—H_2O_2 propellant, M = 2.4, injection perpendicular to nozzle axis.

[¶] A trace of chemical reaction is indicated for the hot vitiated H_2 injection, since experimental I_{sps} values slightly exceed theoretical values (theory is based on injectant being inert). The "potential" for this kind of injectant is far from being achieved in these experiments.

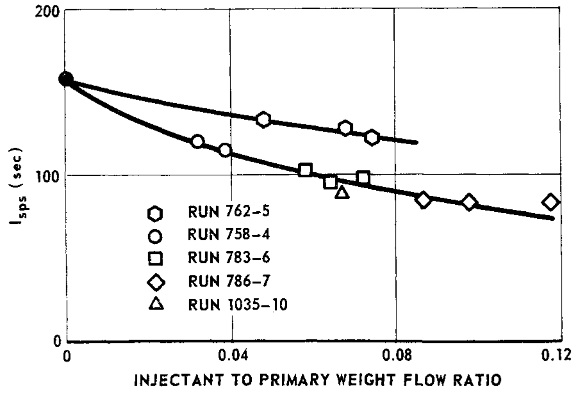

Fig. 5 Freon 12 injection into solid propellant conical rocket nozzle; data of Newton and Spaid[8] (solid point is predicted value).

compare theoretical performance of selected injectants on a common basis.

To do this, we select a hypothetical high-performance rocket motor ($\gamma = 1.24$, $\mathfrak{M} = 22$, $T_0 = 3000°K$)** and injection configuration ($\alpha = 90°$, $M = 2.5$, $T = 1714°K$). Injectant is at 298°K unless it is cryogenic (in which case it is at its boiling point) or propellant gas (with $T_{0g} = 3000°K$). We assume that exhaust gases contain excess H_2 and that chemical reaction between injectant and H_2 can occur in some cases. In practice, a multitude of chemical reactions may occur. The kinetic energy of the liquid jet is based upon no-loss-injection from 1000 psi storage to 14.7 psi; it is always quite small compared to thermochemical energy. (For example, $V_l^2/2 = 1.65$ cal/g for H_2O.) For density considerations, we assume that gases are stored at 1500 psi; however, I_{sps} is computed for sonic injection with $p_j = p$. Without kinetic considerations, it is impossible to define a priori the degree or extent of vaporization, dissociation, or chemical reaction. We have, therefore, tended toward equilibrium thermochemistry, but in some cases more than one mechanism is considered for the same injectant (NH_3, Br_2, N_2O_2, and H_2O_2).

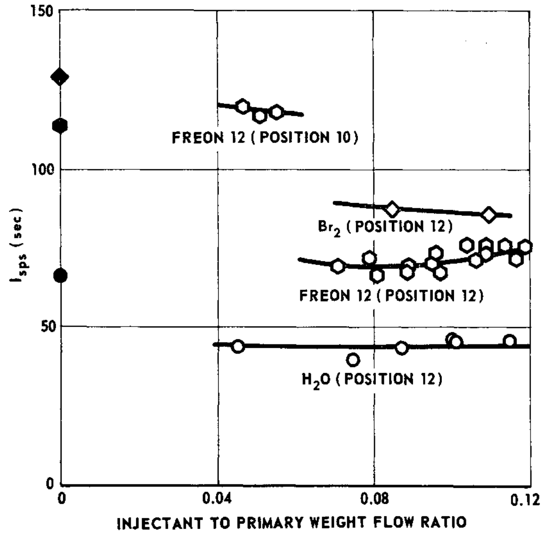

Fig. 6 Liquid-injection data of Green and McCullough[9] (solid points are predicted values).

Table 3 gives results for a variety of injectants in six classes: inert gases, inert liquids, reactive gases (gases that react with H_2-rich propellant exhaust), dissociative liquids, reactive liquids, and liquid bipropellant systems. Mechanisms involving a chemical change are shown in parentheses. Q and ν values are also tabulated. For volume-limited application, the tabulated density impulse I_{ds} (I_{sps} multiplied by injectant density, sec-g/cm^3) is useful. Of course, the necessary hardware associated with the injection system in any particular application must also be considered.

Inert gas injectants other than propellant gases have I_{sps} values that decrease with increasing molecular weight. For gases of practical value, the I_{ds} is less than about 100, although this value might be increased slightly by higher-pressure storage. For reactive gases, I_{sps} values are higher but not enough to offset the poor packaging of gaseous injectants. Gaseous injectants may be preferred where a "ready" simple fluid-injection system is required.

Propellant gas injection is exceptionally good for volume-limited application. Storage of condensed phase reactants is the obvious reason for its exceptional qualities. Similarly, hot-gas injectants derived from other condensed systems have large density impulses. Note the equivalent bipropellant systems (not optimized for mixture ratio) ($I_{ds} = 589$ for N_2H_4-ClF_3 and $I_{ds} = 498$ for N_2O_4-N_2H_4) and the somewhat poorer monopropellants ($I_{ds} = 362$ for H_2O_2 and $I_{ds} = 304$ for N_2H_4). These "hot gas" injectants might well represent the most efficient practical fluid-injectant systems for thrust vector control.

A broad span in I_{ds} values for inert liquids is found. I_{sps} values are usually modest, ranging from 100 to 200. (Liquid H_2, with its low molecular weight is exceptional, with I_{sps} equaling 491, but again its density impulse is poor.) Examples of inert liquids with high density impulses are Br_2 and Hg. In addition Br_2 has the extra potential of reacting with H_2; this improves its performance slightly. Some heavy liquids used in mineralogical analyses may be useful as fluid injectants; for example, acetylene tetrabromide, thallous formate water solutions, and stannic bromide with carbon tetrachloride all have densities near 3 g/cm^3. (We could not locate thermochemical data on these compounds; hence they are not included in Table 3.) Heavy liquids have a distinct advantage over the seemingly superior reactive liquids: their performance depends on heat transfer alone for vaporization and does not rely on chemical reactions.

The reactive liquids—oxidizers that may react with H_2 in the mainstream—are potentially the best. Some of these are considered to be storable, such as ClF_3, N_2O_4—an obvious advantage for missile application. Of the examples considered, ClF_3 is best ($I_{ds} = 767$). One example, a salt solution 50% by weight of $NaClO_4$ in H_2O, is a type of liquid injectant (solution or mixture) that merits further exploration. The practical performance of reactive injectants depends strongly on mixing and chemical kinetics and can be answered only by experiment.

Another important observation from Table 3 that needs emphasizing is: *endothermic decomposition always decreases performance*. Note especially the better performance for incomplete dissociation of N_2O_4 ($N_2O_4 = 2NO_2$ as compared with $N_2O_4 = 2NO + O_2$) and for nondissociated NH_3, compared with dissociated NH_3. Even $Fe(CO)_5$, which speculatively gives 5 moles of gas per mole of injectant, is average. Although dissociation provides additional gas volume, the heat absorbed usually offsets this gain. A mathematical basis for this observation is found in the theory.

The difference between I_{sps} with dissociation and I_{sps} without dissociation is

$$\Delta I_{sps} = \frac{pA}{w}\,\frac{kM^2}{(M^2-1)^{1/2}}\left[-\frac{\Delta H_{DT}}{c_pT} + (\nu - 1)\frac{\mathfrak{M}}{\mathfrak{M}_j}\right]$$

** In the present calculations, it is not necessary to specify the pressure. The motor gases are assumed to be ideal gases with the specified conditions (frozen flow), and the chemistry associated with the injectant is assigned. To be more exact, one should include pressure-dependent chemical equilibrium (dissociation) but we exclude this refinement. The propellant selected for this study has a specific impulse ($p_0/p_e = 1000/14.7$) of about 260 sec.

Table 3 Theoretical effective specific impulse for various injectants in conjunction with a hypothetical rocket motor and nozzle

Mechanism	Q, kcal/mole	ν	I_{sps}, sec	I_{ds}, g sec/cm^3
Inert gases				
H_2	10.34	1	744	5.78
He	7.03	1	593	9.68
N_2	10.98	1	236	26.9
O_2	11.59	1	228	31.3
Ar	7.03	1	236	40.5
Xe	7.03	1	199	106
Ra	7.03	1	193	175
Propellant gases[a]	−13.2	1	434	720[a]
Inert liquids				
H_2O	24.5	1	127	127
CCl_2F_2 (Freon 12)	37.5	1	159	205
$CCl_2F_2CClF_2$ (Freon 114)	43.9	1	155	223
CO_2	18.8	1	171	188
NH_3	13.1	1	208	170
Br_2	20.0	1	173	539
Hg	21.7	1	172	2330
O_2[b]	14.6	1	186	212
H_2[b]	12.4	1	491	34
N_2[b]	13.9	1	192	155
Reactive gases[c]				
O_2 $(=2H_2O - 2H_2)$	−102.3	0	672	92.1
Dissociative liquids				
NH_3 $(= \frac{1}{2} N_2 + 1\frac{1}{2}H_2)$	37.1	2	161	132
N_2O_4 $(= 2NO_2)$	50.3	2	154	223
N_2O_4 $(= 2NO + O_2)$	77.5	3	141	204
H_2O_2 $(= H_2O + \frac{1}{2} O_2)$	6.79	1.5	247	362
N_2H_4 $(= N_2 + 2H_2)$	19.7	3	304	304
Fe $(CO)_5$$(=$ Fe $+ 5CO)$	124	5	163	237
Reactive liquids[c]				
N_2O_4 $(= N_2 + 4H_2O - 4H_2)$	−206	1	478	692
H_2O_2 $(= 2H_2O - H_2)$	−53.2	1	434	635
ClF_3 $(= HCl + 3HF - 2H_2)$	−148.3	2	424	767
Br_2 $(= 2HBr - H_2)$	− 5.97	1	193	602
O_2[b] $(= 2H_2O - 2H_2)$	−105.3	0	586	669
$NaClO_4 + 6.8H_2O$ $(= HCl + Na + 10.8H_2O - 4.5H_2)$[d]	54.4	1.07	222	. . .
Liquid bipropellant				
$N_2H_4 + ClF_3$ $(= N_2 + HCl + 3HF)$	−129.4	2.5	393	589
$2N_2H_4 + N_2O_4$ $(= 3N_2 + 4H_2)$	−166.5	2.33	406	498

[a] Solid propellant density, 1.66 g/cm³.
[b] Cryogenic ($T_l = T_{\text{boiling}}$).
[c] H_2 supplied by rocket exhaust.
[d] Saturated solution.

where ΔH_{DT} is the dissociation energy. It follows that I_{sps} decreases with dissociation if

$$(\nu - 1) < (\mathfrak{M}_j \; \Delta H_{DT})/(\mathfrak{M} \; c_pT)$$

where $\mathfrak{M}_j \; \Delta H_{DT}$ is the molar heat of dissociation and $\mathfrak{M} \; c_pT$ is the molar heat content of the mainstream. $\mathfrak{M} \; c_pT$ approximately equals 18 kcal/mole for the present example and is probably typical. If only a single chemical bond is broken, $\mathfrak{M}_j \; \Delta H_{DT}$ will be the bond energy and $\nu = 2$. Since typical bond energies exceed 18 kcal/mole, the inequality suggests that performance loss occurs with dissociation. Increasing T by injecting nearer the nozzle throat will reduce this loss.

Secondary-injection-induced nucleation, recombination of mainstream species, and other thermochemical processes are tractable with the linearized solution. These items are usually overlooked entirely in other analyses.

Injection angle and port location are constant for the results given in Table 3 ($\alpha = 90°$, $M = 2.5$). These effects have been theoretically studied in detail in an earlier report,[4] and the results can be summarized as follows.

All injectants show similar curves of I_{sps} vs M. I_{sps} has a minimum at $M \approx 2$ and a singularity at $M = 1$; the singularity is usual in linear supersonic flow theory and has little practical value. I_{sps} increases monotonically with increasing $M > 2$. At $M = 5$ (a practical limit) I_{sps} is about 30% larger than at the $M \approx 2$ minimum.

Upstream and normal injection give about the same I_{sps}. Downstream injection gives lower performance and is most significant for gas injection; liquid jet momentum is too small to have much effect (experimental observation of improved performance for upstream liquid injection is probably due to longer residence times, which aid vaporization and mixing).

Conclusions

This linearized analysis for thrust vector control by fluid injection provides a very simple method of computing potential effective specific impulse for any fluid injectant and rocket combination. Aerothermochemical effects are clearly elucidated, and agreement between theory and experiment is quite good. Because time-dependent kinetic phenomena are neglected, the analysis serves only as a culling agent, and experiments are eventually required.

Theoretical calculations of effective density impulse of selected fluid injectants lead to the following conclusions with respect to volume-limited applications.

1) Compressed gases, inert or reactive, are least desirable.

2) Dense inert liquids are superior to light inert liquids or to liquids that undergo endothermic decomposition, but they are inferior to liquids that undergo exothermic decomposition. Some dense inert liquids, such as Br_2 and Hg, have exceptional theoretical merit.

3) Bipropellant injection or injection of liquids that chemically react with the rocket exhaust show the greatest potential.

4) Propellant gas injection (hot gas from rocket chamber) compares well with reactive liquids and bipropellants.

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